RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

SECOND YEAR [BATCH 2017-20]
B.A./B.Sc. FOURTH SEMESTER (January – June) 2019
Mid-Semester Examination, March 2019

Date : 25/03/2019 **MATHEMATICS (Honours)**
Time : 2 pm – 4 pm Paper : IV Full Marks : 50

[Use a separate Answer Book for each group]

Group – A [25 marks]

Answer any two questions from Question Nos. 1 to 3 : [2×9]

1. a) Suppose $\mathcal{P}(\mathbb{N})$ is the power set of $\mathbb{N}$. Define

$d:\mathcal{P}(\mathbb{N})\times\mathcal{P}(\mathbb{N})\to\mathbb{R}$ by (for $A,B\in\mathcal{P}(\mathbb{N})$),

$$d(A,B)=\begin{cases}0, \text{if } A\Delta B=\phi\\ \dfrac{1}{m}, \text{if } m \text{ is the smallest elment of } A\Delta B\end{cases}$$

Verify whether d is a metric on $\mathcal{P}(\mathbb{N})$.

b) Find two closed sets A,B in $\mathbb{R}$ such that $A\cap B=\phi$ and $d(A,B)=0$ where 'd' denotes the usual metric in $\mathbb{R}$. Justify your answer.

c) Show that $\{0,1\}$ is a G_δ –set in $\mathbb{R}$. [4+3+2]

2. a) Define equivalent metrics. Give example with justification.

b) Show that the metric space l_∞ is not separable. [4+5]

3. a) Suppose A be an uncountable subset of $\mathbb{R}$. Show that A has a limit point

b) Define two metrics 'd_1' and 'd_2' on $\mathbb{N}$ such that $(\mathbb{N},d_1)$ has no isolated point and $(\mathbb{N},d_2)$ has only one isolated point. Justify your answer. [3+(3+3)]

Answer any two questions from Question Nos. 4 to 6 : [2×3.5]

4. For $\alpha=(x_1,x_2)$ and $\beta=(y_1,y_2)$ in $\mathbb{R}^2$, $(\alpha|\beta)=x_1y_1-x_2y_1-x_1y_2+4x_2y_2$.

Prove that $\mathbb{R}^2$ forms an inner product space with respect to $(|)$.

5. Let V be the subspace of $\mathbb{R}[x]$ of polynomials of degree at most 3. Equip V with the inner product

$$(f|g)=\int_0^1 f(t)g(t)\,dt \quad \text{for } f,g\in\mathbb{R}[x].$$

Apply the Gram-Schmidt process to the basis $\{1,x,x^2,x^3\}$.

6. Let $(|)$ be the standard inner product on $\mathbb{C}^2$. Prove that there is no non-zero linear operator T on $\mathbb{C}^2$ such that $(\alpha|T\alpha)=0$ for every α in $\mathbb{C}^2$.

Group – B [25 marks]

Answer <u>any two</u> questions from <u>Question Nos. 7 to 9</u> : [2×5]

7. Show that the tangent to the curve $x^3+y^3-3axy=0$ at a point $\left[\neq(0,0)\right]$ where it meets the parabola $y^2=ax$ is parallel to y-axis.

8. Find the equation of the evolute of astroid $x^{2/3}+y^{2/3}=a^{2/3}$.

9. In the curve $r^n=a^n\cos n\theta$, verify that the radius of curvature $\rho=\frac{r^2}{(n+1)p}=\frac{a^n}{(n+1)r^{n-1}}$, where p is the length of perpendicular from the pole upon any tangent to the curve.

Answer <u>any three</u> questions from <u>Question Nos. 10 to 14</u> : [3×5]

10. Find the equation of the integral surface given by the differential equation
$2y(z-3)p+(2x-z)q=y(2x-3)$, which passes through the circle z = 0, $x^2+y^2=2x$.

11. Using Charpit's auxiliary equation, find a complete integral of the equation
$$2(z+xp+yq)=yp^2.$$

12.a) Eliminate the functions f and F from $y=f(x-at)+F(x+at)$.

b) Find the PDE arising from $\phi\left(\frac{z}{x^3},\frac{y}{x}\right)=0$, where ϕ is an arbitrary function of its arguments. [2+3]

13. Find the complete solution of the differential equation $\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=X$, using the method of variation of parameters, where, P,Q, and X are functions of x only. [5]

14. If λ is an Eigen value of Sturn-Liouville problem and ϕ_1 and ϕ_2 are the corresponding eigen functions then prove that ϕ_1 and ϕ_2 are linearly dependent. [5]

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